Physics Practical Laboratory Manual

M. Sc. Physics IInd Semester

Laboratory Name: Advanced Quantum Mechanics Lab

Course Code: PPPBLT2

Credit: 1 (0+0+1)

List of Experiments

Exp. 01. By analyse the Zeeman Effect in mercury vapour, determine the fine structure constant by Fabry-Perot Interferometry. (Experiment)

Exp. 02. Calculate the energy difference between the singlet and triplet state of He Atom. (Mathematical solutions only)

Exp. 03. Two identical particles of spin 1/2 are enclosed in a one-dimensional box potential of length L with walls at x=0 and x=L. Find the Ground state energy. (You can use any programming language)

Experiment:01

Objective:

By analyse the Zeeman Effect in mercury vapour, determine the fine structure constant by Fabry-Perot Interferometry. (Experiment)

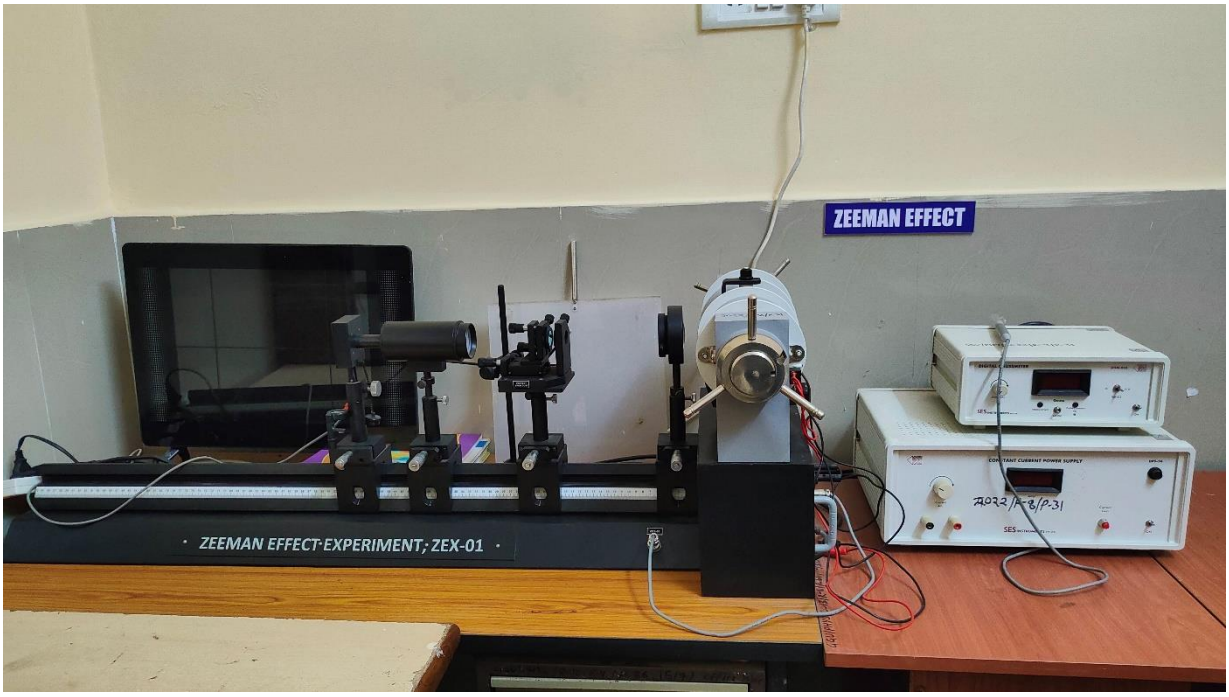
Introduction:

The "Zeeman Effect Experiment" is the splitting up of the spectral lines of atoms when they're placed in a magnetic field. This is one of the few fundamental atomic physics experiments which can be performed in a teaching laboratory.

In 1896 it was observed by Zeeman that, when an atom is placed in an external magnetic field, and then excited, the spectral lines it emits in the de-excitation process are split into several Components. The Zeeman effect clearly indicates space quantization leading to splitting of electronic energy levels of the atom into several components in the presence of an external magnetic field. The basic unit of splitting is the Bohr Magneton which contains charge to mass ratio of the electron, i.e. e/m.

In order to realize this experiment in optical region, suitable atoms are chosen whose electronic energy levels lie in visible region of light. One such atom is mercury. A mercury source is taken. When the mercury vapor is heated (excited) the atoms emit light of several wave lengths. To focus on a particular wavelength (say green light 546.1 nm) a monochromator is used to filter out other colours (wave lengths). The electronic energy levels are resolved in the form of rings whose diameter corresponds to the energy of emitted radiation with the help of a Fabry Perot Etalon. The rings with and without the application of magnetic field can be seen by naked eye. Since these are very small and closely spaced, a CCTV camera is employed to obtain large and well-spaced rings on a TV screen. The measurement of the diameter of these rings with and without the external field will give the information about the energy of electronic energy levels of the atom. In the present of the external magnetic field, the light from the mercury source is polarized.

In the present set-up, the observations can be taken both in the transverse and in the longitudinal geometry with respect to the direction of the external magnetic field. We use the transverse geometry for the measurement of diameter of the rings leading to the calculation of Bohr Magneton. The polarization of the light is analysed/ verified both in the transverse as well as in the longitudinal geometries.



Description of Apparatus

Experimental Set-up for Zeeman Experiment

The set-up consists of the following:

1. Mercury Discharge Tube, MT-01 placed in the gap of tapered pole pieces Electromagnet. EMU-50T powered by Constant Current Power Supply, DPS-50

2. Black Shield with Slit

3. Monochromator, Narrow Band Interference Filter, IF-01

Central Wave Length: 546nm

T_{max}: 74%

HBW: 8 nm

4. Polarizer PL-01 fitted with Single Focussing Lens, FL-1

5. Quarter Wave Plate

6. Fabry Perot Etalon, FP-01

7. Telescope with Focussing Lens, Triplet, FL-3

8. Optical Bench: OB-1

9. CCD Camera600 (High Resolution CCD Camera)

10. Monitor 17 3TV - 17

11. Digital Gaussmeter. DGM-102

Theory

In general, an atom will have a total magnetic dipole moment, μ , due to the orbital and spin magnetic dipole moments, μ_{l1}, μ_{l2}, . . . and μ_{s1}, μ_{s2}, . . . of its optically active electrons. The other electrons are in completely filled sub-shells which have no net magnetic dipole moments. When this magnetic dipole moment of the atom is in an external magnetic field B, it will have the usual potential energy of orientation

$$\Delta E = -\mu . B \quad . \qquad (1)$$

Each of the atom's energy levels will be split into several discrete components corresponding to the various values of ΔE associated with the different quantized orientations of μ relative to the direction of B. Let us evaluate μ in terms of orbital and spin magnetic moments of optically active electrons expressed in terms of their respective orbital and spin angular moment.

$$\begin{aligned}
\mu &= (\mu_{l1} + \mu_{l2} + ...) + (\mu_{s1} + \mu_{s2} + ...) \\
&= \left(-\frac{g_l \mu_b}{\hbar} L_1 - \frac{g_l \mu_b}{\hbar} L_2 - .....\right) + \left(-\frac{g_s \mu_b}{\hbar} S_1 - \frac{g_s \mu_b}{\hbar} S_2 - .....\right) \\
&= -\frac{g_l \mu_b}{\hbar}(L_1 + L_2 + ...) - \frac{g_s \mu_b}{\hbar}(S_1 + S_2 + ...) \\
&= -\frac{\mu_b}{\hbar}[(L_1 + L_2 + ...) + 2(S_1 + S_2 + ...)] \quad .
\end{aligned}$$

Here $L_1, L_2, ...$ are orbital angular momenta and $S_1, S_2, ...$ spin angular momenta of active electrons, $\mu_b = e\hbar/2mc$ is Bohr magneton , and g_l and g_s are orbital and spin g factors. These factors have values $g_l = 1$ and $g_s = 2$. If the atom obeys LS coupling, the individual orbital angular momenta couple to give the total orbital angular momentum L, and the individual spin angular momenta couple to give total spin angular momentum S. The expression for the total magnetic dipole moment of the atom now simplifies to

$$\mu = -\frac{\mu_b}{\hbar}[L + 2S] \quad . \qquad (2)$$

Note that the total magnetic dipole moment of the atom is not antiparallel to its total angular momentum

$$J = L + S \quad . \qquad (3)$$

This has come about because of different values of the orbital and spin g factors. The result is that the behavior of μ is quite complicated. It precesses about J with a precessional frequency which is proportional to the strength of the internal magnetic field of the atom. The result is that the

average value of μ, which is the component μ_J of μ along the direction of J enters in Eq.(1) for ΔE

$$\Delta E = -\mu_J . B \qquad (4)$$

The component μ_J is given by

$$\begin{aligned}
\mu_J &= \left(\frac{\mu . J}{J^2}\right) J \\
&= -\frac{\mu_b}{\hbar}\frac{(L + 2S).(L + S)}{J^2} J \\
&= -\frac{\mu_b}{\hbar}\left(\frac{L^2 + 2S^2 + 3L.S}{J^2}\right) J \\
&= -\frac{\mu_b}{\hbar}\left(1 + \frac{J^2 + S^2 - L^2}{2J^2}\right) J \\
&= -\frac{\mu_b}{\hbar} g_{LSJ} J \quad , \qquad (5)
\end{aligned}$$

as $\quad L.S = \left(J^2 - L^2 - S^2\right)/2 \quad .$

The factor g_{LSJ} is called Lande's $g-$factor and is given by

$$\begin{aligned}
g_{LSJ} &= 1 + \frac{J^2 + S^2 - L^2}{2J^2} \\
&= 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)} \quad , \qquad (6)
\end{aligned}$$

where l, s and j are respectively the values of orbital, spin and total angular momenta in the state characterized by L, S and J.

The potential energy of interaction in an external magnetic field B is now given by

$$\begin{aligned}
\Delta E &= -\left(-\frac{\mu_b}{\hbar} g_{LSJ}\right) J.B \\
&= \frac{\mu_b}{\hbar} g_{LSJ} J_Z B \\
&= \mu_b B g_{LSJ} m_J \quad , \qquad (7)
\end{aligned}$$

taking z-axis along B and $J_Z = m_J \hbar$.

Thus an energy level characterized by L, S and J will get split up into $(2j + 1)$ sublevels corresponding to $m_J = -j, -(j-1), -(j-2), \ldots\ldots, (j-1), j$ with energy separation between the adjacent sublevels given by $\mu_b B g_{LSJ}$.

An optical transition can take place between any two levels provided

$$\begin{aligned}
\Delta l &= \pm 1 \\
\Delta j &= 0, \pm 1 \quad \text{but not} \quad j = 0 \rightarrow j = 0 \\
\Delta m_J &= 0, \pm 1 \quad .
\end{aligned}$$

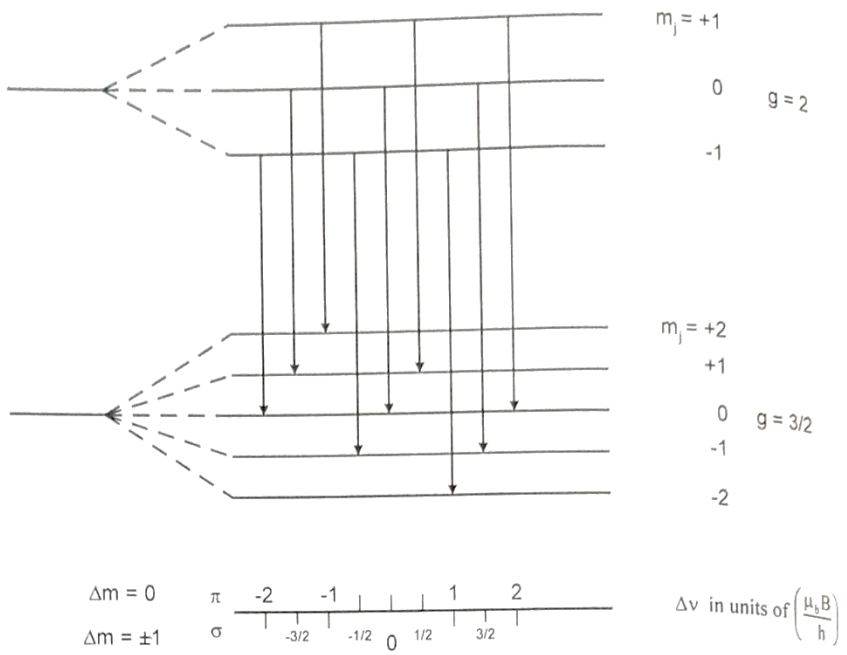


Fig. 1 : Structure of the Zeeman multiplet arising in a transition from a $^3S_1$ to a $^3P_2$ level; the mercury green line at 5461A is an example of such a transition.

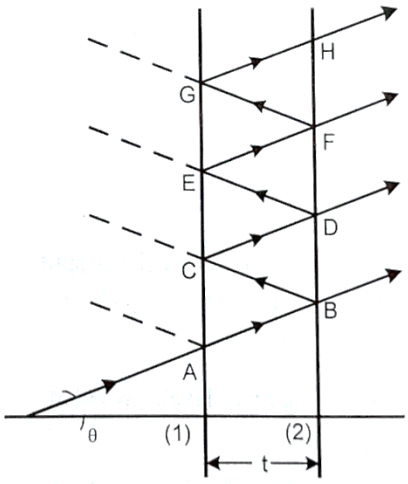


Fig. 2 : Reflected and transmitted rays at the two parallel surface (1) and (2) of a Fabry Perot etalon. The etalon spacing is t.

Let us consider the transition corresponding to the 546.1 nm prominent green line of mercury spectrum with which we are concerned here. This line arises from a transition between the $^3S_1(6s7s)$ state to the $^3P_2(6s6p)$ state. Figure 1 shows the energy-diagram for these two states without and with a magnetic field. The upper level gets split into three corresponding to m_J = 1, 0 and -1 , and the lower into five corresponding to m_J = 2 , 1 , 0 , -1 and -2 . The g-factors for the upper and lower states are

$$^3S_1(J=1, L=0, S=1) \qquad g_{lsJ} = 2$$
$$^3P_2(J=2, L=1, S=1) \qquad g_{lsJ} = 3/2$$

As shown in Fig.1 , in the presence of the magnetic field, the 546.1 n.m. line gives rise to nine components. This is because of different g-factors for the initial and final states. This is a case of the (*so called*) anomalous Zeeman effect.

When viewed in the transverse geometry, (i) the first group where $\Delta m_J = -1$ gives σ-lines whose light is polarized perpendicular to the magnetic field, (ii) the middle group where $\Delta m_J = 0$ gives π-lines whose light is polarized parallel to the direction of the field and (iii) the last group where $\Delta m_J = 1$ gives σ-lines whose light is again polarized perpendicular to the magnetic field.

In the longitudinal geometry where the light beam is along the direction of the magnetic field, the beam corresponding to π-lines which is polarized along the direction of the magnetic field (has electric vector along the direction of the magnetic field) cannot travel as light waves are transverse electromagnetic waves. The beam corresponding to π-lines is therefore not observed. The beam corresponding to six σ- lines is circularly polarized when viewed in the longitudinal direction, the three components corresponding to $\Delta m_j = -1$ as right circularly polarized while those corresponding to $\Delta m_j = 1$ as left circularly polarized.

The shift in frequencies of these nine lines is indicated in the lower part of Fig.1 in units of $\mu_b B/h$.

We are interested in the three π-lines for the measurement of Bohr Magneton and use transverse geometry. These are picked out by using a polarizer whose pass-direction is kept parallel to the direction of the magnetic field. The σ- lines which are polarized perpendicular to the magnetic field get blocked. The frequencies of these three π - lines are

$$\nu_0 - \frac{\mu_b B}{2h}, \quad \nu_0, \quad \nu_0 + \frac{\mu_b B}{2h} . \qquad (8)$$

This frequency shift $\Delta\nu = \mu_b B/2h = eB/8\pi m$ is measured in this experiment. As this is very small, a high resolution device, a Fabry Perot etalon, based on multiple beam interferometry, is used.

The Fabry Perot etalon consists of two optically flat (to within about 20 nm) glass plates, coated on the inner surface with a partially transmitting metallic layer (reflection coefficient ≈ 0.95) (Fig. 2). The outer surface is slightly inclined (about 0.1°) with respect to the inner one, to avoid multiple reflections, which give rise to "ghost" fringes. The plates are assembled in a holder (Fig. 3) and held apart (with spacing t) by three very accurately machined spacers. Three spring-mounted screws are used to apply pressure, and by careful adjustment, the plates are made parallel.

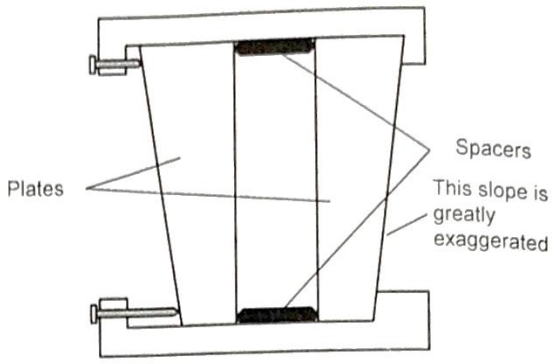


Fig. 3: Mounting of the interferometer plates into a Fabry Perot etalon. Note that the slight slope of the two sides of a plate is usually of the order of 1/10 degree

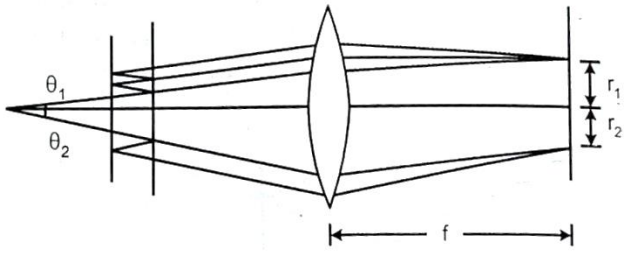


Fig. 4: Focusing of the light emerging from a Fabry-Perot etalon. Light entering the etalon at an angle θ is focused onto a ring of radius r=fθ where f is the focal length of the lens.

An *almost* parallel beam, from an extended source at the focus of a lens, falls on the etalon. The emerging parallel rays *B, D, F,* etc are brought to a focus by the use of a good quality lens of focal length f as shown in Fig. 4. Light entering the etalon at an angle θ is focused onto a ring of radius $f\theta$. When θ satisfies the condition

$$2t\cos\theta = n\lambda ,$$

with n an integer, a bright ring will appear in the focal plane with radius being given by

$$r_n = f\theta_n$$

The order n_0 corresponding to the interference at the centre is $2t/\lambda$. n_0 is in general *not* an integer.

Now

$$n_0 = \frac{2t}{\lambda} = \frac{n}{\cos\theta_n} = \frac{n}{\left(1-\theta_n^{\,2}/2\right)}$$

since θ_n is small. This leads to

$$\theta_n = \sqrt{\frac{2(n_0-n)}{n_0}}$$

and

$$r_n^{\,2} = \left(\frac{2f^2}{n_0}\right)(n_0-n) . \qquad (9)$$

The order n_1 of the first bright ring counting from the centre is less than n_0 since $n_1 = n_0\cos\theta_1$. Let us take $n_1 = n_0 - \varepsilon$, with fractional order ε lying between zero and one. In general for the p^{th} ring of the pattern as measured from the centre

$$n_p = (n_0 - \varepsilon) - (p-1)$$

or

$$n_0 - n_p = (p -) + \varepsilon . \qquad (10)$$

From Eqs. (9) and (10) we obtain for square of the radius of the p^{th} ring

$$r_p^{\,2} = \left(\frac{2f^2}{n_0}\right)(p-1+\varepsilon) \qquad (11)$$

and the difference between the squares of the radii of adjacent rings

$$r_{p+1}^{\,2} - r_p^{\,2} = \frac{2f^2}{n_0} . \qquad (12)$$

This difference is constant. The squares of the radii of successive rings are linearly related and form an arithmetic progression.

Now if there are two components of a spectral line (in the present experiment we have three components) with wavelengths λ_a and λ_b very close to one another, their fractional orders at the centre will be given by

$$\varepsilon_a = \frac{2t}{\lambda_a} - n_1(a) = 2t\bar{\nu}_a - n_1(a)$$

$$\varepsilon_b = \frac{2t}{\lambda_b} - n_1(b) = 2t\bar{\nu}_b - n_1(b) .$$

Here, $\bar{\nu}_a$ and $\bar{\nu}_b$ are wave numbers, and $n_1(a)$ and $n_1(b)$ are orders of the first rings. As the wavelengths are very close, $n_1(a) = n_1(b)$. The difference in wave numbers between the two components is therefore

$$\bar{\nu}_a - \bar{\nu}_b = \frac{\varepsilon_a - \varepsilon_b}{2t} . \qquad (13)$$

The fractional order ε can be obtained using Eq. (11):

$$\frac{r_{p+1}^2}{r_{p+1}^2 - r_p^2} - p = \varepsilon \qquad (14)$$

Consider a line which has three components (as in the present experiment) a, b, c, and let the respective radii be $r_{1a}, r_{2a}, r_{3a}, \ldots$, for component a; $r_{1b}, r_{2b}, r_{3b}, \ldots$, for component b and similarly for component c. From Eq.(12), it is clear that the difference between the squares of the radii of any two adjacent rings of component a,

$$\Delta_a = r_{(p+1),a}^2 - r_{p,a}^2 = \frac{2f^2}{n_{0,a}}$$

is equal (to within a very small amount) to the similar difference for component b,

$$\Delta_b = r_{(p+1),b}^2 - r_{p,b}^2 = \frac{2f^2}{n_{0,b}}$$

or any other component of the same line. We shall take the average of these. Let these differences be designated by Δ. Eq.(14) now leads to

$$\varepsilon_a = \frac{r_{(p+1),a}^2}{\Delta} - p$$

$$\varepsilon_b = \frac{r_{(p+1),b}^2}{\Delta} - p$$

$$\varepsilon_c = \frac{r_{(p+1),c}^2}{\Delta} - p ,$$

and the required separation (in wave numbers) between the two components, a and b, using Eq.(13), is given by

$$\Delta\bar{\nu} = \frac{\varepsilon_a - \varepsilon_b}{2t} = \frac{r_{p,a}^2 - r_{p,b}^2}{2t\Delta} . \qquad (15)$$

The difference $\delta_{a,b}^p$ between the squares of the radii of the p^{th} rings of components a and b is found to be independent of p. We shall take the average of these. Let these be designated by δ_{ab}. The required separation between the two components is finally given by

$$\Delta\bar{\nu}_{ab} = \frac{\delta_{ab}}{2t\Delta} . \qquad (16)$$

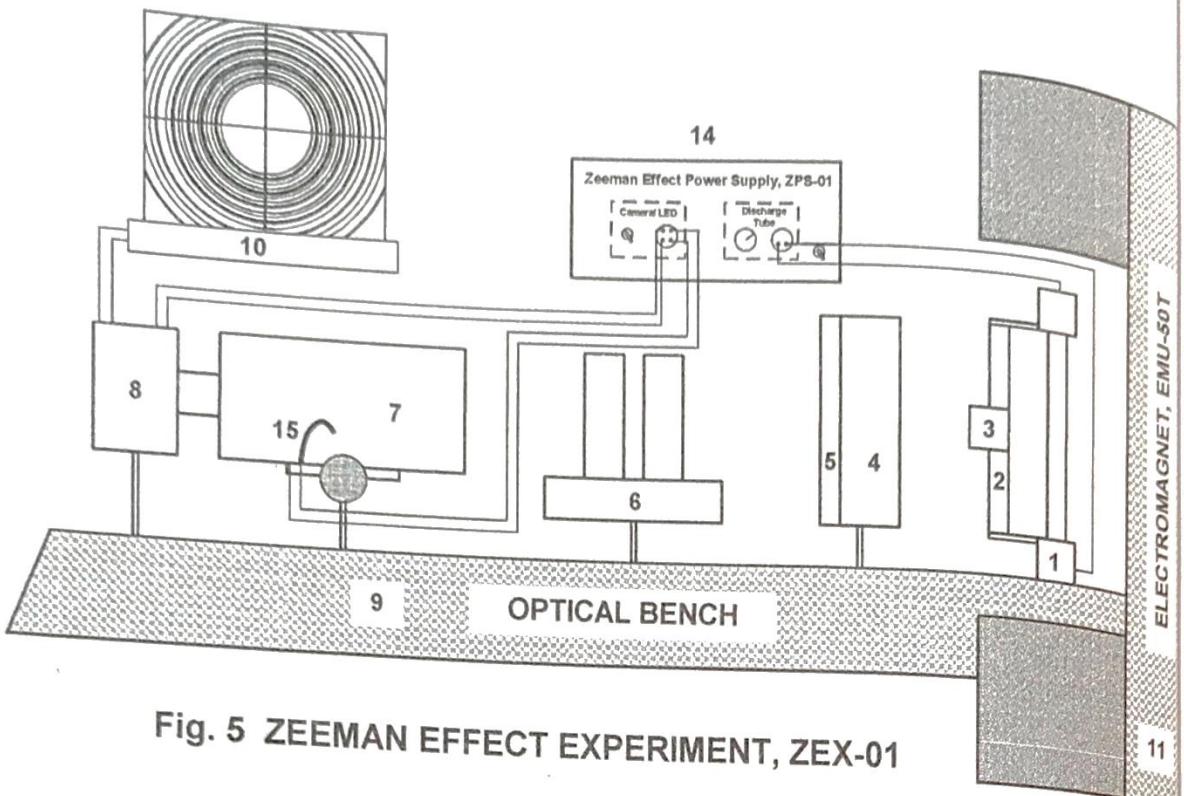
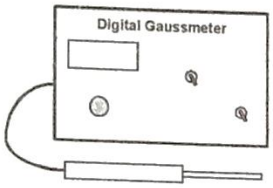


Fig. 5 ZEEMAN EFFECT EXPERIMENT, ZEX-01



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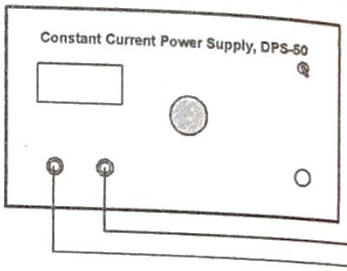

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1. Mercury Discharge Tube, MT-01
2. Black Shield
3. Monochromator, IF-01
4. Focussing Lens, FL-01
5. Polariser, PL-01
6. Fabrey Perot Etalon, FPE-01
7. Telescope with Focussing Lens Triplet, FL-3
8. CCD Camera, CAM-700
9. Optical Bench, OB-1
10. TV Monitor, TV-14
11. Electromagnet, EMU-50T
12. Constant Current Power Supply, DPS-50
13. Digital Gaussmeter, DGM-102
14. Zeeman Effect Power Supply, ZPS-01
15. LED Torch Light

As the result depends on the ratio δ_{ah}/Δ, the dimensions used in measuring the radii of the ring system or the amplification of the interference pattern do not matter.

Using this wave number separation in Eq.(8) for the Zeeman line splitting, we get

$$\Delta\nu = \frac{\mu_b B}{2h} = \frac{eB}{8\pi m} = c\Delta\bar{\nu} = \frac{c\,\delta}{2t\,\Delta}$$

or

$$\frac{e}{m} = \frac{8\pi c}{B}\left(\frac{\delta}{2t\,\Delta}\right). \qquad (17)$$

Further the **Bohr Magneton** μ_b is given by $\mu_b = \frac{2hc}{B}\left(\frac{\delta}{2t\,\Delta}\right)$

Average values Δ and δ are required

Procedure:

(1) Calibration of EMU-50V

(a) Adjust spacing between pole pieces using space provided.
(b) The space between the pole piece should be centered along the electromagnet center line marked in white.
(c) Take out the Hall Probe of the Gaussmeter from its casing and switch ON the unit, keeping the probe away from electromagnet or any other magnetic material. Adjust the Gaussmeter reading to Zero by "Zero Adj." knob
(d) Place the DGM-102 Hall Probe along the centre of EMU-50 using the Probe Holder provided. The alignment of probe should be parallel to the face of the pole pieces, le. perpendicular to the magnetic field.
(e) Check the sign in Gaussmeter. In case it shows-ve sign, turn the direction of Hall Probe by 180
(f) Take the reading of Gaussmeter at zero current. It corresponds to the residual magnetic field of electromagnet.
(g) Now slowly increase the DPS-50 current in small steps and make a table of current vs. magnetic field upto a maximum current of 4A.
(h) Plot a graph between the current and the magnetic field. It will be used later in the calculation.

(2) Setting up of Experiment

(a) Remove the gaussmeter probe from electromagnet.

(b) Place Mercury Discharge Tube using the Holder provided, aligning the same with the center of electromagnet. The position of light emitting hole should face the narrow band filter on the optical bench.

(c) Place the set of converging lens and polarizer as near to the filter as possible. (

d) Set the distance between Etalon plate holder to near 8mm (using the spacer provided). This will make the effective distance between the Etalon plates as 4mm, which is a part of calculation.

(e) To obtain fringes

a) Place the components on the optical bench as shown in Fig. 5

b) Adjust the height of all the components as is done in other optical experiments e.g. Nodal Slide/Bi-prism so that the overall optical axis lies in the same straight line.

c) Move No. 6 and No. 7 further away so that it becomes convenient to peep through etalon with naked eye.

d) While observing through the etalon you may see multiple images of the slit as shown in Fig. 6.

e) While observing the slit images, adjust the two screws provided on the etalon VERY CAREFULLY so that the images collapse to a single slit.

1) As soon as it is achieved, the fringes will be seen even with naked eye.

g) Bring No. 6 and No. 7 closer now and adjust No. 7 by moving back and forth to get clear fringes on the TV screen.

h) Adjust the two screws on the etalon further (VERY CAREFULLY) to obtain the fringes in the centre of the TV screen.

(3) Taking observations

a) Set the center of the specified line at the cross section as marked on your TV screen.

b) Apply maximum magnetic field. Visible splitting of spectrum lines will appear on the TV screen.

c) Adjust the polarizer to focus the fringes.

d) Bring the inner most splitting of the 1st spectrum line at the cross section and note the micrometer reading. This position is Ri.

e) Now move the micrometer to adjust the second spectral splitting of the 1" line at the cross wire and note its reading.

f) Similarly take readings of splitting’s in different spectral lines.

g) Complete the observation table as given.

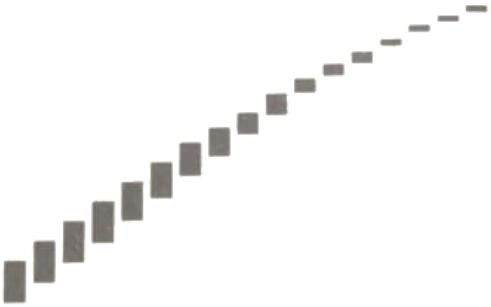

Fig. 6

Observations & Tabulations

A. Measurement of Bohr Magneton:

The setup field direction transverse geometry, that is, the light beam is perpendicular magnetic

Thickness of air gap for Etalon t = 5mm = 0.5cm

Magnetic Field H=11.70 KGauss

Micrometer initial reading R_i = 7.81mm

No. of spectral line	Splitting of spectral lines	Split position R_s	Actual radius $R=(R_s-R_i)$	R^2 (mm)
1	a b c			
2	a b c			
3	a b c			
4	a b c			

Average values of Δ and δ can be determined as shown below:

$$\Delta_{12}^{a} = R_{a2}^{2} - R_{a1}^{2} = - - - - - - - - -$$
$$\Delta_{12}^{b} = R_{b2}^{2} - R_{b1}^{2} = - - - - - - - - -$$

$$\Delta_{12}^{c} = R_{c2}^{2} - R_{c1}^{2} = - - - - - - - - -$$

Mean $\Delta_{12} = (\Delta_{12}^{a} + \Delta_{12}^{b} + \Delta_{12}^{c})/3$

Calculate for 3 & 4 spectral lines and mean…….

Average value of $\Delta = (\Delta_{12} + \Delta_{34})/2 =$

$$\delta_{ab}^{1} = R_{b1}^{2} - R_{a1}^{2} = - - - - - - - - -$$

$$\delta_{bc}^{1} = R_{c1}^{2} - R_{b1}^{2} = - - - - - - - - -$$

$$\delta_{ab}^{2} = R_{b2}^{2} - R_{a2}^{2} = - - - - - - - - -$$

$$\delta_{bc}^{2} = R_{c2}^{2} - R_{b2}^{2} = - - - - - - - - -$$

$$\delta_{ab}^{3} = R_{b3}^{2} - R_{a3}^{2} = - - - - - - - - -$$

$$\delta_{bc}^{3} = R_{c3}^{2} - R_{b3}^{2} = - - - - - - - - -$$

$$\delta_{ab}^{4} = R_{b4}^{2} - R_{a4}^{2} = - - - - - - - - -$$

$$\delta_{bc}^{4} = R_{c4}^{2} - R_{b4}^{2} = - - - - - - - - -$$

Average value of $\delta = (\delta_{ab}^{1} + \delta_{bc}^{1} + \delta_{ab}^{2}$----------)/8=

Calculation:

Average value of Δ=

Average value of δ=

$$\bar{\nu} = \frac{\delta}{2t\Delta} = \quad - - - - - - - - - - - - - - m^{-1}$$

$$\mu_B = \frac{2hc}{B}\Delta\bar{\nu} = \quad - - - - - - - - - - - -$$

Bohr Magneton μ_B=

Results:

Experiment:02

Objective:

Calculate the energy difference between the singlet and triplet state of He Atom. (Mathematical solutions only)

Students must write the complete He atom's wave function while taking spin and spatial variation into consideration. They must demonstrate that in the He atom, the triplet state has less energy than the singlet state by taking into account the Hamiltonian for a two-electron system and appropriately applying time-independent perturbation theory.

Experiment:03

Objective:

Two identical particles of spin 1/2 are enclosed in a one-dimensional box potential of length L with walls at x=0 and x=L. Find the Ground state energy. (You can use any programming language)

The single electron wave-function of an electron in a one-dimensional box must be taken into consideration by students, who must also assume that the two-electrons wave-function is simply the product of a single wave function. Without taking spin into account, students attempt to compute the ground state energy by taking into account the proper Hamiltonian and applying the time independent perturbation theory. Students attempt to compute the analytical solution first, and if possible, they could even write a program.